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ABSTRACT

This study was conducted to determine if an integrated content-methods, two-course sequence for preservice elementary teachers (PSTs) affects the mathematical beliefs of these students in three main areas: (1) beliefs about mathematics as a discipline; (2) beliefs about how mathematics is learned and should be taught; and (3) beliefs about themselves as learners and teachers of mathematics. Three sets of questionnaires were administered to 37 PSTs before and after completion of integrated mathematics/contents courses which included field experiences. Two students were also interviewed. Results indicated some modifications of PSTs' beliefs about the nature of mathematics following the courses since they perceived it as less rule-oriented and dependent upon memorization; were less likely to see math in totally right-wrong, one answer-one method terms; and held a different view of the importance and nature of word problems. Significant changes also occurred in personal teaching efficacy which was attributed directly to the field experiences. (Contains 39 references.) (Author)

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Tracing Mathematical Beliefs of Preservice Teachers Through Integrated Content-Methods Courses

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This study was conducted to determine if an integrated content-methods, two-course sequence for preservice elementary teachers (PSTs) affects the mathematical beliefs of these students in three main areas: 1) beliefs about mathematics as a discipline, 2) beliefs about how mathematics is learned and should be taught, and 3) beliefs about themselves as learners and teachers of mathematics. Three sets of questionnaires were administered before and after the PSTs completed the courses which included field experiences. Interviews were also conducted for more in-depth data and possible insights into specific experiences that served as catalysts in belief alteration. Results indicated some modification of PSTs' beliefs about the nature of mathematics since they perceived it as less rule-oriented and dependent upon memorization; were less likely to see math in totally right-wrong, one answer-one method terms; and held a different view of the importance and nature of word problems. Significant changes also occurred in personal teaching efficacy which was attributed directly to field experiences.

In recent years, investigations of mathematical performance have considered more than just knowledge, facts, concepts, and procedures. It is now recognized that control decisions and processes (Garofalo & Lester, 1985), beliefs about the nature of mathematics (Schoenfeld, 1987), and attitudes and other affective variables (McLeod, 1991; Reyes, 1984) have tremendous effects on mathematical performance. Students often have conceptions about the subject matter they study and themselves that affect the decisions they make in learning mathematics and ultimately in their mathematical achievement. Likewise, the nature of teachers' beliefs about the subject matter and about its teaching and learning may well play a significant role in shaping their instructional practices (Barr, 1988; Cooney, 1985; Grant, 1984; Stodolsky, 1985; Thompson, 1984).

Fennema (1989) and Fennema and Franke (1992) have proposed models to guide research on learning behaviors and the development of teachers' knowledge. In each model, the development of autonomous learning behavior for students and the contextual development of teachers' knowledge, including content knowledge,

pedagogical knowledge and knowledge of learners' cognitions, are heavily influenced by internal beliefs. This implies that the things the teacher says and does, the beliefs and expectations held by the teacher, and the activities in which learners are expected to participate are all ways in which teachers influence students' internal beliefs and learning behaviors.

Ball (1987) identified five dimensions of teacher beliefs: 1) beliefs about mathematics, 2) beliefs about learning mathematics, 3) beliefs about pupils as learners and "doers" of mathematics, 4) beliefs about teaching mathematics, and 5) beliefs about learning to teach (or getting better at teaching) mathematics. A person's conception about the nature of mathematics as a discipline may be viewed as that person's conscious or unconscious beliefs, concepts, meanings, rules, mental images and preferences concerning mathematics. These subject matter beliefs, which constitute a rudimentary philosophy of mathematics, have been shown to be significant factors in the learning of mathematics and in influencing teacher behaviors (Bassarear, 1986; Cooney, 1985; Erlwanger, 1975; Ernest, 1988; Lester, Garofalo, & Kroll, 1989; Thompson, 1984). This dimension of beliefs about mathematics encompasses how someone would answer such questions as: What is mathematics? What kind of knowledge is it? What do mathematicians do? How important is mathematics? As Schoenfeld (1985) and Lester, Garofalo, and Kroll (1989) point out, students' beliefs about mathematics can play a dominant, often overpowering role in their problem solving behavior.

What teachers or students consider to be desirable goals of studying mathematics, their own roles as students or teachers, appropriate classroom activities and emphases, and acceptable outcomes of instruction are all part on one's conception of mathematics learning and teaching. Research indicates that differences in teachers' beliefs about the nature of mathematics itself appear to be related to differences in their views about mathematics teaching (Cobb, Yackel, & Wood, 1989; Cooney, 1985; Thompson, 1984). A strong relationship has also been observed

between teachers' conceptions of teaching and their conceptions of students' mathematical knowledge (Cobb, Wood, & Yackel, in press)

Most studies of changes in student or teacher mathematical beliefs have focused only on a content or a pedagogical dimension and the context in which it was affected. The purpose of this study was to determine if a Taylor University two-course sequence for preservice elementary teachers, which integrates both content and pedagogical knowledge, affects the mathematical beliefs of these students. These general mathematical beliefs were grouped into three main areas:

1. beliefs about mathematics
2. beliefs about mathematics learning and teaching
3. beliefs about self as a learner of mathematics

The impetus for this investigation came from an increasing awareness of acute differences among the preservice teachers in conceptions of mathematics and self-confidence in learning and teaching mathematics. Informal observations over several years indicated that the attitudes and beliefs that PSTs brought to the classroom, as well as their knowledge and skills, were affecting their mathematical behavior and achievement.

Beyond the primary question of which of the students' beliefs were affected by the course experiences, a secondary consideration, assuming that belief changes occurred, was to try to determine some specific course experiences that were major catalysts in altering beliefs. In general, answering these questions will provide information about how a PST's beliefs might be altered over a period of time and what experiences might have the most impact on those beliefs.

Description of the Courses

All Taylor University elementary education majors must complete two 5-semester-hour courses, Math 201 and Math 202. The first course includes, in part, the mathematical topics of the number system through the real numbers (numeration systems, number

bases, whole, integer, rational, etc.), probability, and statistics. The second course emphasizes geometry, measurement, spatial topics, and problem solving. In addition, special attention is paid to concrete teaching aids (Dienes, 1967), laboratory methods (Cathcart, 1977), and classroom pedagogy based on various learning theories (Bruner, 1986; Piaget, 1973), and use of calculators and computers (NCTM, 1989,1991).

Through the integrated structure of the courses, the goal is to provide students with opportunities to increase the depth of their knowledge of topics appropriate for the elementary and middle schools and simultaneously to examine sound pedagogical practices for teaching those topics to children. The underlying rationale for such a program structure is its more natural ability to integrate students' content and pedagogical content knowledge rather than trying to artificially separate them.

The format of the classes involves much small group work and discussions with an emphasis on doing mathematics through a problem solving approach. Many problems and activities from the T104 class developed for preservice teachers at Indiana University are incorporated into the courses (Lester, Maki, LeBlanc, & Kroll, 1992). The classes meet in a mathematics laboratory in which students have ready access to a wide-variety of materials to use in solving problems and to use in teaching mathematics to children during their field experiences. An important component of the courses is the heavy emphasis on field experience work. Each student is responsible for teaching a math-lab type lesson in a local elementary school classroom once a week. This practice teaching is done in teams of two or three and provides PSTs with opportunities to write lesson plans, prepare materials, receive feedback on teaching, and conduct self-evaluations.

In summary, an important objective of the program is to produce teachers who are reflective decision makers in the classroom. A vital ingredient in that process is the conscious effort to help preservice teachers become aware of and examine their own mathematical beliefs. The goal of many course components

is, therefore, to provide opportunities for students to confront and challenge their current mathematical beliefs. The following are some of the major features of the Math 201-202 sequence:

*readings of journal articles such as Arithmetic Teacher and written personal reflections of ideas presented,
*mathematical problem solving experiences, particularly with nonroutine problems,
*small group discussions and cooperative problem solving (2-4 persons),
*whole class discussions of mathematical concepts encountered,
*frequent use of manipulative materials for modeling mathematical ideas and concepts,
*viewing and evaluating ideas of classroom practice presented in videos,
*frequent contact with elementary students and teachers through continuing field experiences,
*planning lessons, teaching, and writing evaluations of personal teaching experiences.

METHOD

Subjects

The subjects of this study were preservice elementary education teachers enrolled in Math 201 and then Math 202 at Taylor University, a small (enrollment 1800) private liberal arts college in north-central Indiana. Thirty-seven students were enrolled in Math 201 but only 27 of these students (22 female, 5 male) completed Math 202 the following semester. There were 3 freshman, 10 sophomores, and 14 juniors, all of whom had previously passed a university-administered mathematics proficiency examination required of all majors. The average score on the SAT math section for all in-coming university freshman the previous year was 526, and, in general, the overall academic profile of the students would place them above the national average.

The Instruments

Three different beliefs questionnaires were administered to the PSTs. These questionnaires contain Likert-type, multiple-choice questions, as well as some open-ended questions, dealing with a variety of mathematical beliefs.

The *Indiana Mathematics Belief Scales* (Kloosterman & Stage, 1992) consist of six, 6-item scales intended to measure students' motivational beliefs in these areas:

1. I Can Solve Time-Consuming Mathematics Problems
2. There Are Word Problems That Cannot be Solved with Simple, Step-by-Step Procedures
3. Understanding Concepts is Important in Mathematics
4. Word Problems are Important in Mathematics
5. Effort Can Increase Mathematical Ability
6. Mathematics is Useful in Daily Life

Each of these constructs is thought to be related to motivation and thus to achievement on mathematical problem solving.

The second questionnaire was an abbreviated and slightly altered version of a questionnaire developed and used by Schoenfeld in a study of high school students (Schoenfeld, 1989). The form used in this study consisted of 43 closed and 9 open questions designed to assess students' perceptions about mathematics and school practice, their views of school mathematics, English, and social studies, motivation, and personal and scholastic performance and motivation (see Appendix).

The third questionnaire was used to measure beliefs about mathematics teaching. It was developed at Vanderbilt University (Witherspoon & Shelton, 1991) and addresses pedagogical, content, and curricular issues. Its five constructs measured by separate Likert-type subscales on the instrument are:

1. Sense of Personal Mathematics Teaching Efficacy
2. Sense of Universal Mathematics Teaching Efficacy
3. Beliefs about Elementary School Mathematics Content
4. Beliefs about Elementary School Mathematics Pedagogy
5. Beliefs about Learning Processes

Because the reliability was low on subscales 3, 4, and 5 of this

questionnaire (all had a Chronbach's alpha less than 0.50), statistics on these constructs were not computed.

Data Collection

Both quantitative and qualitative data were collected during the study. The three belief questionnaires were administered to all students during the first week of first semester and to the same students enrolled in Math 202 during the last week of second semester. Statistics are reported only on those students who completed both courses (n = 27). Means and standard deviations as well as paired sample t-tests for differences in scores between the beginning and end of the school year were computed on the responses to the various scales.

In addition, two students were interviewed during the last week of school. These individuals (both female) were chosen because they represented extremes in their initial mathematical knowledge and background and because they had the ability to clearly express themselves. The first student, Amy, appeared confident and received an A on the first Math 201 test while the second, Susan, expressed much apprehension about mathematics and received an F on the first exam, These students were each interviewed once for a 30-45 minute session which was tape recorded.

Finally, data were collected from analysis of students' written work, particularly reflective papers and field experience reports, from students' oral comments, and from observations of students' problem-solving and teaching behaviors.

RESULTS

Initial Beliefs

Overall results from the questionnaires administered before students took the mathematics courses reveal that the preservice teachers had a narrow, fairly restricted view of mathematics. Beliefs of the preservice teachers, as measured by the *Indiana*

Mathematics Belief Scales (IMBS), were neutral to positive on all scales. The scores were lowest on the STEPS subscale (mean=18.04) and highest on the UNDERSTANDING subscale (mean=25.22). Students were evidently convinced of the usefulness and importance of mathematics in everyday life as shown by the USEFULNESS subscale (mean=24.37). Results are summarized in Table 1 on the next page.

Results from items on the Schoenfeld-based questionnaire (see Appendix) indicated that the PSTs began the Math 201-201 sequence with a limited view of the discipline. The responses to items 1,4,5,9,22,24,25, and 52 show that the students generally saw mathematics, at least as learned in school, as mostly facts and procedures to be memorized and applied in exactly the one correct way to arrive at the only correct answer to a problem. More than half (sixteen) of the respondents to item 52 believed that memorization was "very important" while ten stated that memorization was "somewhat important." Several students indicated that "understanding concepts and applications" or deductive reasoning and problem solving skills were also important. One student noted that although memorization was important, it "shouldn't be if you understand" and another stated that one could "memorize and still have trouble solving problems."

The PSTs of this study, as did Schoenfeld's high school students, believed in native ability in English and social studies, but most strongly in mathematics (items 8,12,16) They also saw mathematics as more dichotomized into "completely right or completely wrong" answers than either English or social studies (items 9,13,17).

Belief Changes

Although the mean scores for each scale of the IMBS (Table 1) increased from the first to second administrations, the PSTs' beliefs appear to be relatively stable over this time period. After computing paired-sample t-tests comparing scores on the two administrations of the test, only two scales showed significant

Table 1.
Means, Standard Deviations, and Paired Sample t-tests for Differences in IMBS Scores Between the Beginning of Math 201 and the End of Math 202

		Beginning of Math 201		End of Math 202		
Scale	N	Mean	(SD)	Mean	(SD)	t
EFFORT	27	21.44	3.33	21.74	2.78	0.41
WORD PROBLEMS	27	19.81	3.04	22.48	3.13	3.27*
STEPS	27	18.04	3.61	22.26	3.21	4.67#
UNDERSTANDING	27	25.22	2.31	26.22	2.18	1.60
DIFFICULT PROBLEMS	27	19.56	4.30	19.74	4.29	0.16
USEFULNESS	27	24.37	2.78	25.41	2.79	1.48

*p < .01

#p < .001

differences, STEPS and WORD PROBLEMS.

Further results from questionnaire 2 (Appendix), consistent with the previous data, indicate that some beliefs held by the PSTs were modified during the year. These include significant differences on items dealing with the nature of mathematics and how it is learned (sections 3,4,8). Items 9,21,22,23,24, and 25 indicate significant modifications in student mathematical beliefs. Also, differences in responses to open questions in section 8 indicated some shifts in student beliefs about what is involved in learning and understanding mathematics (items 49,51,52). Because of the ambiguity of the items in sections 1 and 2 and the differences in interpretation of the questions by respondents, no t-test scores were computed for these questions.

As noted earlier, comparative statistics were computed on only two subscales of the *Elementary School Mathematics Teaching* (ESMT) *Beliefs Inventory* (questionnaire 3 - see Table 2). No significant difference was found on the Universal Teaching Efficacy subscale - the extent to which one believes that teaching in general has a positive influence on student achievement in elementary school math, ($t = 1.32$). However, a significant change did occur on the Personal Teaching Efficacy subscale - the extent to which one believes that one's own mathematics teaching can have a positive effect on students' achievement in elementary school mathematics ($t = 5.08$, $p < .001$, $n = 27$). Both dimensions of teaching efficacy have been shown to influence the teaching-learning process (Ashton & Webb, 1986; Gibson & Dembo, 1984; Kelly, 1987).

TABLE 2

ELEMENTARY SCHOOL MATHEMATICS TEACHING BELIEFS INVENTORY

Subscale		Fall		Spring		
	N	mean	SD	mean	SD	t
Personal Teaching Efficacy	27	33.81	5.83	37.37	5.54	5.08*
Universal Teaching Efficacy	27	32.41	2.96	33.26	3.51	1.32

* $p < .001$

Some significant belief changes about appropriate mathematics classroom practices were evidenced by analysis of responses to individual items on the ESMT. The PSTs were more convinced of the importance of problem solving in the elementary curriculum after completing the math courses (items 32,45,46), $t = 3.41$, $p<.01$. They more strongly believed that children were able to invent algorithms (item 36), $t = 4.95$, $p<.001$. The PSTs were more inclined to integrate calculator use into elementary math (items 4,11), $t = 6.62$, $p<.001$, and to use manipulative materials throughout elementary grades (items 16,35), $t = 5.20$, $p<.001$. As teachers, they also were more inclined to pose open-ended questions (more than one reasonable solution) to their pupils (items 12, 14), $t = 6.01$, $p<.001$.

DISCUSSION

The neutral position of the PSTs on the STEPS subscale of the IMBS indicates that many students were not aware that in "real" mathematics, many problems cannot be solved by simple, step-by-

step-procedures. This inclination to always search for prescribed algorithms and memorized procedures is consistent with other studies (Ball, 1990; Frank, 1985; Lindquist, 1990; Raymond, Santos, & Masingila, 1991; Schoenfeld, 1989) and is due, at least in part, to their limited exposure to nonroutine problems in their prior mathematical experience. Further evidence of this limited perspective is seen in the WORD PROBLEMS scale, which had one of the lowest means on the first administration. This initial neutral position of the PSTs (mean = 19.81) again reveals little inclination to view mathematics as something more than computational skills. Interestingly, these two dimensions of STEPS and WORD PROBLEMS received the lowest mean scores of the six scales when administered to the Indiana University PSTs, the original study sample (Kloosterman, 1992).

Despite the emphasis by students on memorization, there was a recognition of the importance of understanding. This belief in the importance of understanding concepts is shown in the results on the UNDERSTANDING scale of the IMBS which had the highest mean (25.22) of the six scales. This positive outcome is again entirely consistent with the results of the Indiana University preservice teachers in the original study (Kloosterman, 1992). The emphasis on the one hand on rules, computation, and memorization as keys to doing mathematics and, on the other hand, strong emphasis on the importance of understanding mathematical concepts and viewing mathematics as a creative discipline where one can "discover things by yourself" (item 21, questionnaire 2) seem to be contradictory beliefs. This discrepancy may be between what students think should be true about math and what they actually do in learning school math. These results seem to support Schoenfeld's suggestion that many students have come to separate school mathematics - that which they know and experience in their classrooms - from abstract mathematics, the discipline of creativity, problem solving, and discovery, about which they are told but which they have not experienced, or perhaps, have experienced only in out-of-school contexts (Schoenfeld, 1989). If this is so, the positive rhetoric

that students espouse and perhaps believe will unfortunately count for little in problem-solving situations or instructional practices if their behavior is determined more by their experiences than by their professed beliefs.

The results on the USEFULNESS scale (Table 1) are in line with national outcomes (NAEP) where seventh-graders ranked mathematics as the most important academic subject and eleventh graders ranked it as one of the two subjects high in importance (Lindquist, 1990). However, it may be that individuals tend to view math as important and useful in society but less so for them personally. On questionnaire 2, item 48, responses showed that students had only a surface understanding of how math is useful. A few thought they had "never used" or "used very little" the algebra and geometry they had learned. A majority said had used it in solving daily problems and several noted they had used math in other classes or it had increased their reasoning/logical thinking skills. Specific references to usefulness were almost always examples such as buying, selling, money, rent, checkbook, etc. which dealt with essentially the use of computational skills. These PSTs seemed to have progressed only slightly in their limited views of the uses of mathematics from the elementary children studied by Kloosterman and Cougan (1991) whose references were to jobs, sports, or future schooling.

The significant gains on the STEPS and WORD PROBLEMS scales indicate that the frequent encounters with word problems, particularly nonroutine problems, in their mathematics courses persuaded students that mathematics was more than merely computational skills and that stepwise algorithms are often inadequate for solving some problems. This change in perspective was also borne out in student interviews. Amy, a freshman who received an A in both courses, commented on her previous experiences with mathematics:

> *I was always so frustrated in math because...all it seemed like it was was rules and whenever I wanted to know why, people just said, if I asked for help or*

> *something, it was just always, "it's just the way you do it, just take it." And that carried all the way through high school. So, I never really liked math much. It didn't seem like there were reasons behind it; it was just the way you did it.*

Explaining her change in how she viewed mathematics, Amy noted:

> *Before this year, I just saw it as rules so I just saw it as strictly memorizing and now I think I try to look at the whole picture more and looking for patterns and things like that... I used to get so frustrated with word problems. I just could not stand it any longer. Because I was really just trying to figure out the formula. I didn't try to do anything else. And it just, oh, I couldn't stand it. I couldn't stand being wrong. It's hard for me to get out of that mindset - like not all the time, Amy, are you going to be able to just whip those out (quickly apply a rule or formula to get the answer).*

Another student, Susan, who had attended a Montessori school through fifth grade and a private high school, failed the first exam and struggled throughout the first semester. Her recollections of elementary math were flash cards, races, and memorizing facts, all of which she enjoyed and excelled in. No problem solving was included in the elementary curriculum, according to her recollections. In high school, she believed she received As because once she was shown how to do something she easily memorized it. However, she noted that she "couldn't do story problems!" Susan's work improved throughout the year and she earned a B+ for the second semester. She expressed a change also in her view of math:

> *It's not just memorizing numbers. It includes problem solving. I have a different perspective now.*

It is not surprising that changes in students' beliefs as measured on the USEFULNESS and UNDERSTANDING scales did not reach statistical significance because scores were quite high after the initial administration of the scales. It may also be the case that Math 201/202 course experiences did little to challenge beliefs in these areas. It is relevant to note here, however, that terminal responses on open question 48 from questionnaire 2 included not

just everyday computational needs, as were in the initial responses, but also students mentioned problem solving as a useful aspect of studying mathematics - particularly the ability to "look at things from more than one perspective." Also eight students included the ability to teach math as a practical use for the math they had learned, showing perhaps that the PSTs were becoming more focused in their specific career goals and attempting to integrate learning experiences to those objectives.

By the end of the second semester of math courses, the prospective teachers appeared to view math as less of an exact, absolute discipline - free of ambiguity of interpretation with few chances for creative work. By the end of Math 202, students were more inclined to believe that real mathematics problems could be solved by common sense and reasoning rather than knowing school-learned rules (item 23; $t = 5.61$, $p<.001$). They also expressed a stronger belief that mathematics allows one to be creative and discover things on one's own (item 21; $t = 2.62$, $p<.05$) and were less inclined to see memorization of formulas as the best way to do mathematics (item 25; $t = 3.64$, $p<.01$). Solving math problems appeared to the PSTs to be more open to interpretation and context-related than they had previously thought and they exhibited less of the simple right-wrong, one answer-one method dualistic thinking than had appeared earlier ([item 9; $t = 5.20$, $p<.001$], [item 22; $t = 3.26$, $p<.01$], [item 24; $t = 3.40$, $p<.01$]). Responses to item 52 showed students were less inclined after second semester to say memorization was "very important" (3 students vs. 16 students). Most thought memorization of formulas for tests and "basics" was still "somewhat" important but many also mentioned understanding concepts, problem solving methods, or the importance of creative/analytical thinking and intuition in learning mathematics.

Analysis of the data also reveals some important beliefs the students held about themselves as learners. The responses to the items in sections 6 and 7 on questionnaire 2 paint a picture of college students who are motivated to do well in school and see hard work as a necessary ingredient. They believe it is quite

important for them to do well in math (item 43) and try to learn mathematics primarily to do well in the courses which are required for their program (items 30,31). They also see math as interesting (item 32) and having value for helping them think more clearly in general (item 29). At the same time, there is concern about appearing "dumb" to others as a learner of mathematics (item 35). and a lack of self-confidence indicated by a tendency to feel stupid if they don't understand something (item 34). On the average, they were B math students in high school and above average gradewise in college (items 39,40). Compared to others, these students see themselves as above average in mathematical ability and as working at it slightly more than most others (item 42). No significant changes occurred in responses to these items during the study.

Likewise, there were no significant changes on the DIFFICULT PROBLEMS or EFFORT scales of the IMBS, both of which are related to students' self-confidence and inclination to study the subject. For all the experience the PSTs had in working with problem solving, their self-confidence in doing nonroutine or time-consuming problems remained unchanged and neutral. Because students were no more convinced that effort and hard work can make one better at doing mathematics (consistent with their belief in native ability), the less able students particularly may have little motivation for committing much time and energy to the study of mathematics.

There seemed to be great individual differences in this area of self-confidence in learning mathematics. Susan still attributed one's success in math to being "born that way - having those talents and abilities," although she added that one "can work at it and become better." After completing the courses, she rated her confidence level for learning new mathematics as "about two or three (on a scale of 1-10). In contrast, Amy rated her confidence level for leaning new mathematics as "a 7 or 8" and added that she:

> *still struggles with confidence but has improved a lot. Now I'm more willing to try. I think it has to do with,*

> *for one thing, the attitude you have going into it and the way you look at math. If you don't have good self-esteem, you don't think you're a good student and you go into it thinking it's all these rules to memorize, you're not going to try. You think you are going to fail if your answer is always the wrong answer and it's never right. You just stop trying after awhile. I think it's your attitude, learning to look at the big picture - critical thinking has a lot to do with it (being successful). And just having perseverance. Just having patience to keep working at things and not giving up on the first time you don't get it.*

The responses to item 49 on questionnaire 2 may indicate some modification in the PST's conceptions about how mathematics can be learned. Initially, only 6 people believed that students could discover mathematics on their own, while some thought only "basic" math could be discovered and many were not sure. These students seemed more inclined to believe math could be figured out by pupils after completing Math 201-202. No one responded that math had to be shown to students although some qualified their answers with comments such as "can discover elementary math", or "depends on the person." Perhaps, their experiences of solving problems and learning new concepts individually or in small groups (without direct intervention from the teacher) influenced the PSTs view of the learning process.

Some shifts in how the PSTs viewed what it means to understand math were implicated in responses to item 51 on the same questionnaire. On the first semester administration, typical responses were: "I just know", "...can solve it smoothly without help", "no mistakes:, or "can do variations of the same problem" (as in textbook practice problems). None of those responses appeared on the second administration of the questionnaire. At that time, responses were:

> *I know why I'm doing something.*
> *Have conceptual understanding; it makes sense.*
> *Can do related problems or apply it.*
> *I get the right answer.*
> *I can explain it to someone else (verbally or written).*

Amy reflected on the connection between one's view of mathematics

and the learning process:

> *I would say the way I learn, in this year, has changed because I used to see it (math) as rules. I just strictly tried to memorize. But through actually learning myself some this year, I can pick up - I usually understand the concept. I pick up the concepts quickly but I have to keep going over it to really get it... I've found a lot more ways to help me learn, a lot more strategies to help me learn through the year - things that help me that I can pull on when I need them.*

At least for some students, beliefs about the process of learning and understanding mathematics seems to have evolved over the course of the school year.

Although, by the end of the year, the PSTs were no more confident of the power and influence of teachers ***in general*** to influence students' achievement (universal teaching efficacy), they were confident in ***their own*** ability to help students learn mathematics (personal teaching efficacy). This positive change in the PST's self-confidence in teaching math was further evidenced in their written and verbal comments as well. When rating their confidence level (at the end) for 1) being able to explain mathematical concepts to children, 2) planning an appropriate mathematics lesson, and 3) in general, teaching an elementary math class, both Amy and Susan responded with marks from 8 to 10 (out of 10). Both commented that they felt comfortable in a classroom, no longer were frightened to teach math, and felt equipped with necessary skills. Amy explained:

> *My confidence in being able to teach has improved a lot. (The biggest reason) is understanding why things are the way they are. You can't just tell kids, "That's the way it is." I had no idea coming into Math 201 how to teach math - only to teach rules. I found what I was looking for.*

This dramatic shift in confidence in teaching math was attributed, by the students who were interviewed, to the weekly field experiences and to better understanding of the content. Susan noted that her self-confidence in teaching came from

> doing it - preparing good, creative lesson plans and seeing that it worked. The students enjoyed it and did well. My confidence increased with the positive feedback from students.

The public school experiences allowed the PSTs to "put into practice what you're learning." Likewise, Amy commented that she saw the "new methods" she had learned actually worked in the classroom and was very encouraged. From these classroom experiences, "I learned to be organized, prepared, flexible - to handle various situations - and saw that the kids enjoyed it and learned."

Responses relative to beliefs about instructional strategies and practices of the c ssroom teacher showed significant differences in several areas (questionnaire 3). It is likely that these shifts in beliefs about appropriate instructional strategies, (e.g. using calculators and manipulative materials, posing open-ended questions, the importance of problem solving) occurred as a result of reading about such practices, viewing them in use in classrooms on videotapes, trying some of them in their field experiences, and having them modeled in the classroom instructional procedures of Math 201-202.

At least in the short run, certain mathematical beliefs of these PSTs seemed to have undergone modification as a result of their experiences in the integrated content-methods courses and accompanying field experience component during the year. Some beliefs about the nature of mathematics were called into question by course experiences which allowed them to examine their own conceptions about what it means to do and to understand mathematics. Particularly, work with nonroutine problems has the potential for altering students' view of what mathematics is and of the process of problem solving. It is also evident, however, that many of the mathematical beliefs of preservice teachers are fairly stable and resistant to change.

It is important to note that studies have indicated that not only do teachers' beliefs and pedagogical content knowledge influence their classroom practice, but that the relationship also

holds in the reverse direction (Cobb, Yackel, & Wood, 1991). The experiences of interacting with students in the classroom strongly influence teachers' pedagogical content knowledge and beliefs. The relationship between teachers' knowledge and beliefs and their practice appears to be dialectical. Therefore, it may be highly ineffective to attempt to modify prospective teachers' beliefs by designing experiences and interventions for them outside the context of classroom practice. This perspective would indeed imply that a significant influence on the PST's pedagogical beliefs would come from their classroom teaching experience. Perhaps it was when the prospective teachers actually encountered problematic situations in the classroom, that they began to reflect on their own knowlege, beliefs, and practices and to become open to alternative ideas and approaches as they searched for answers to their own questions.

CONCLUSIONS

The implementation of nontraditional teaching methods and experiences in integrated content-methods courses for PSTs and the involvement of the PSTs in an extensive practice teaching component hold promise for modifying their beliefs about content, learning, and pedagogy, and building their self-confidence in teaching mathematics. The modification of negative, counterproductive beliefs relative to each of the areas mentioned is an appropriate affective goal in the preparation of teachers.

Several interesting and important questions remain from this study. An attempt to determine which specific experiences accounted for expressed belief changes in the PSTs was largely unsuccessful. With one exception, the students themselves were unable to identify with much certainty which activities precipitated modifications in their mathematical beliefs. For example, Susan accounted for her belief changes as "having thought about" her experiences. She was frustrated by doing poorly (at the beginning) and "thought about what I thought math was" and also

talked with others about her experiences. Amy's changes came from "almost everything we did - it's hard to sort out." She did mention the impact of cooperative group work and the modeling of innovative teaching practices by the instructor as factors in changing her perspectives. In addition, the content of the courses, specifically heavy emphasis on problem solving and the inclusion of ideas and topics such as patterns, tessellations, spatial thinking, and logical reasoning, helped her to change her ideas about what mathematics is. But because beliefs change slowly over time it is difficult to pinpoint specific factors from the multitude of experiences involved in a year's course work. Most likely, the outcomes of belief changes were a result of a complexity of interactive factors rather than any simple, linear relationship. In any event, a key component appeared to be the students' conscious effort to reflect in specific ways on their experiences.

The one exception to students' uncertainty of the origins of their belief changes involved students' sense of personal mathematics teaching efficacy. The completion of a successful practice teaching experience was cited as the major impetus in building the PSTs' self-confidence. However, since not all students had an equally successful field experience, we might wonder about the effects of a "less than successful" practice teaching experience on a student's personal teaching efficacy. Further research is needed to give us a more detailed analysis of the impact of the early field experience.

In addition, further research on the interactions among beliefs about content and pedagogy and teaching efficacy would be valuable since mastery of content and pedagogy may be only loosely linked to a sense of teaching efficacy. For example, preservice teachers' high degree of teaching efficacy may only reflect an overconfidence in their ability to teach elementary mathematics. Their teaching performance may exhibit serious flaws in content and pedagogy and therefore their high level of teaching efficacy becomes counterproductive.

Another issue that remains to be examined is the strength and stability of students' belief changes. The students responses represent only a picture of their "expressed beliefs" at one point in time. We do not know if these are "primary" beliefs or "derivative" ones that may be less likely to reflect behavioral changes in the future (Cooney, 1985). The ultimate test will be the instructional decisions and behaviors in the classroom when the PSTs become practicing teachers.

Another phenomenon that seemed evident in the analysis of the data was the dramatic change that occurred over the course of the year in mathematical beliefs by some individuals and not by others. This feature of the data is hidden when looking at only group statistics but nevertheless is revealed by comparing responses of individuals before and after completing the integrated courses. Some individuals showed dramatic differences despite the lack of significant mean differences for the group. What aspects of the courses or characteristics of these students resulted in such diverse outcomes? The influence of these experiences on individual belief systems was definitely not the same for all PSTs.

The role of research into mathematical beliefs can be an important one in furthering our knowledge of student achievement and effective instruction. Although this sample was small, the results of the study provide evidence that certain classroom experiences, as included in the integrated content-methods courses described here, have the potential for challenging and perhaps altering some mathematical beliefs of prospective teachers. Especially for students who begin their teacher training with negative, almost debilitative beliefs about mathematics, the examination and modification of those beliefs is a desirable affective goal of instruction and may have far-reaching effects on future elementary pupils. More research is needed to understand how and under what circumstances beliefs change and for whom or in what situations mathematical beliefs will influence behavior.

REFERENCES

Ashton, P.T., & Webb, R.B. (1986). *Making a difference: Teachers' sense of efficacy and student achievement*. Longman, New York.

Ball, D.L. (1987). Dimensions of teacher beliefs. Paper presented at the National Council of Teachers of Mathematics Research Presession, Anaheim, CA.

Ball, D.L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal*, 90(4), 449-466.

Barr, R. (1988). Conditions influencing content taught in nine fourth-grade mathematics classrooms. *Elementary School Journal*, 88, 387-411.

Bassarear, T. (1986). Attitudes and beliefs about learning about mathematics, and about self which most seriously undermine performance in mathematics courses. Paper presented at the New England Educational Research Organization, Rockport, Maine.

Bruner, J. (1986) *Actual minds, possible worlds*. Cambridge: Harvard University Press.

Cathcart, W.G. (ed.). (1977). *The Mathematics Laboratory*. National Council of Teachers of Mathematics, Reston, VA.

Cobb, P., Yackel, E. & Wood, T. (1989). Young children's emotional acts while engaged in mathematical problem solving. In McLeod, D. and Adams, V (Eds.) *Affect and Mathematical Problem Solving*. Springer-Verlag, New York.

Cobb, P., Yackel, E., & Wood, T. (1991). Curriculum and teacher development: psychological and anthropological perspectives. In Fennema, Carpenter, & Lamon (Eds.). *Integrating Research on Teaching and Learning Mathematics*. SUNY Press, Albany.

Cobb, P., Yackel, E., & Wood, T. (in press). Learning through problem solving: A constructivist approach to second grade mathematics. In E. von Glasersfeld (Ed.), *Constructivism in mathematics education*. Dordrecht, Holland: reidel.

Cooney, T.J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*, 16, 324-336.

Dienes, Z.P. (1967). *Building Up Mathematics*. Hutchinson Educational Ltd., London.

Erlwanger, S. (1975). Benny's conceptions of rules and answers in IPI mathematics. *Journal of Mathematical Behavior*, 1(2), 7-25.

Ernest, P. (1988, July). The impact of beliefs on the teaching of mathematics. Paper prepared for ICME VI, Budapest, Hungary.

Fennema, E. (1989). The study of affect and mathematics: a proposed generic model for research. In McLeod D. & Adams, V. (eds.), *Affect and Mathematical Problem Solving*. Springer-Verlag, New York.

Fennema, E. & Franke, M. (1992). Teachers' knowledge and its impact. In Grouws, D. (ed.) Handbook of Research on Mathematics Teaching and Learning. National Council of Teachers of Mathematics, Reston, VA.

Frank, M. (1985). Mathematical beliefs and problem solving. Unpublished doctoral dissertation, Purdue University, West Lafayette.

Garofalo, J., & Lester, F.K. (1985). Metacognition, cognitive monitoring and mathematical performance. *Journal for Research in Mathematics Education*, 16, 163-176.

Gibson S., & Dembo, M. H. (1984). Teacher efficacy: A construct validation. *Journal of Educational Psychology*, 76 (6), 42-48.

Grant, C.E. (1984). A study of the relationship between secondary mathematics teachers' beliefs about the teaching-learning process and their observed classroom behaviors. Doctoral dissertation, University of North Dakota. *Dissertation Abstracts International*, 46, DA8507627.

Kelly, T.A. (1987). Teachers' sense of efficacy: clarification, assessment, and predictive validity. Unpublished master's thesis. Vanderbilt University, Nashville, TN.

Kloosterman, P. (1992). *Preparing elementary teachers to teach mathematics, a problem-solving approach: Final report, Vol. V: research and evaluation component.* Bloomington: Indiana University, Mathematics Education Development Center. (ERIC Document Reproduction Service No. ED 349184)

Kloosterman, P. & Cougan, M. (1991). Students' beliefs about learning elementary school mathematics. Paper prepared for AERA, Chicago.

Kloosterman, P. & Stage, F. (1992). Measuring beliefs about mathematical problem solving. School Science and Mathematics, 92(3), 109-115.

Lester, F.K., Maki, D.P., LeBlanc, J.F., & Kroll, D.L. (1992). *Preparing elementary teachers to teach mathematics: a problem-solving approach, Final report Vol. II, content component.* Bloomington: Indiana University, Mathematics Education Development Center. (ERIC Document Reproduction Service No. ED 349 181)

Lester, F., Garofalo, J., & Kroll, D. (1989). Self-confidence, interest, beliefs, and metacognition: key influences on problem-solving behavior. In McLeod, D., & Adams, V. (eds.) *Affect and Mathematical Problem Solving.* Springer-Verlag, New York.

Lindquist, M.M. (Ed.) (1990). *Results from the fourth mathematics assessment.* Reston, VA: National Council of Teachers of Mathematics.

McLeod, D. (1991). Research on learning and instruction in mathematics: the role of affect. In Fennema, Carpenter, & Lamon (eds.) *Integrating Research on Teaching and Learning Mathematics.* SUNY, Albany.

National Council of Teachers of Mathematics, (1989). *Curriculum and Evaluation Standards for School Mathematics.* The Council, Reston, VA.

National Council of Teachers of Mathematics, (1991). *Professional Standards for Teaching Mathematics.* The Council, Reston, VA.

Piaget, J. (1973). *To Understand is to Invent: The Future of Education.* Viking, New York.

Raymond, A., Santos, V., & Masingila, J. (1991). The influence of innovative instructional processes on mathematical belief systems. Paper presented at AERA, Chicago.

Reyes, L. (1984). Affective variables and mathematics education. *Elementary School Journal*, 84, 558-581.

Schoenfeld, A. (1985). Mathematical problem solving. New York: Academic Press.

Schoenfeld, A. (1987). What's all the fuss about metacognition? In A.H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education*, 20(4), 338-355.

Stodolsky, S.S. (1985). Telling math: Origins of math aversion and anxiety. *Educational Psychologist*, 20, 125-133.

Thompson, A. (1984). The relationship of teachers conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15, 105-127.

Witherspoon, M. & Shelton, J. (1991). Measuring elementary school teachers beliefs about teaching mathematics: A preliminary report. Paper presented at Association of Teacher Educators, New Orleans.

APPENDIX
BELIEFS QUESTIONNAIRE 2
(Adapted from Schoenfeld, 1989)

SECTION 1 The math that I learn in school is...	Very true	Sort of true	Not very true	Not at all true	Fall Mean	Spring Mean	Signi-ficant t
1. mostly facts and procedures that have to be memorized.	1	2	3	4	1.70	1.81	
2. thought provoking.	1	2	3	4	1.89	2.23	
3. just a way of thinking about space, numbers and problems.	1	2	3	4	2.59	2.54	
SECTION 2 When a teacher asks a question in math class...	**Always**	**Usually**	**Occas-ionally**	**Never**	**Fall Mean**	**Spring Mean**	**Signi-ficant t**
4. you have to remember the right answer to answer it correctly.	1	2	3	4	2.41	2.31	
5. there are lots of possible right answers you might give.	1	2	3	4	3.15	2.77	
6. you have to think really hard to answer it.	1	2	3	4	2.37	2.35	
7. the students who understand only need a few seconds to answer correctly.	1	2	3	4	2.26	2.23	
SECTION 3	**Very true**	**Sort of true**	**Not very true**	**Not at all true**	**Fall mean**	**Spring mean**	**Signi-ficant t**
8. Some people are good at math and some people just aren't.	1	2	3	4	1.63	1.88	
9. In mathematics something is either right or it's wrong.	1	2	3	4	1.74	2.77	5.208#
10. Good mathematics teachers show students lots of different ways to look at the same question.	1	2	3	4	1.50	1.04	
11. Good math teachers show you the exact way to answer the math questions you'll be tested on.	1	2	3	4	2.56	2.77	
12. Some people are good at English and some just aren't.	1	2	3	4	1.88	2.00	
13. In English something is either right or wrong.	1	2	3	4	2.42	2.04	
14. Good English teachers show students lots of different ways to look at the same question.	1	2	3	4	1.44	1.92	

* P< .01
\# P< .001

SECTION 3 CONTINUED	Very true	Sort of true	Not very true	Not at all true	Fall Mean	Spring mean	Signi-ficant t
15. Good English teachers show you the exact way to answer the English questions you'll be tested on.	1	2	3	4	2.56	2.58	
16. Some people are good at Social Studies and some just aren't.	1	2	3	4	2.35	2.16	
17. In Social Studies something is either right or it's wrong.	1	2	3	4	2.15	2.16	
18. Good Social Studies teachers show lots of different ways to look at the same question.	1	2	3	4	1.52	1.62	
19. Good Social Studies teachers show you the exact way to answer the questions you'll be tested on.	1	2	3	4	2.59	2.69	
SECTION 4	**Very true**	**Sort of true**	**Not very true**	**Not at all true**	**Fall mean**	**Spring mean**	**Signi-ficant t**
20. Everything important about mathematics is already known by mathematicians.	1	2	3	4	2.81	3.08	
21. In mathematics you can be creative and discover things by yourself.	1	2	3	4	1.81	1.38	2.62+
22. Math problems can be done correctly in only one way.	1	2	3	4	1.81	3.36	3.26*
23. Real math problems can be solved by common sense instead of the math rules you learn in school.	1	2	3	4	2.72	2.00	5.61#
24. To solve math problems you have to be taught the right procedure or you can't do anything.	1	2	3	4	2.27	3.04	3.40*
25. The best way to do well in math is to memorize all the formulas.	1	2	3	4	2.59	3.15	3.64*
SECTION 5 When you get the wrong answer to a math problem...	**Very true**	**Sort of true**	**Not very true**	**Not at all true**	**Fall mean**	**Spring mean**	**Signi-ficant t**
26. It is absolutely wrong-there's no room for argument.	1	2	3	4	2.59	2.73	
27. You only find out when it's different from the book's answer.	1	2	3	4	2.67	2.50	
28. You have to start all over in order to do it right.	1	2	3	4	2.56	2.54	

*p< .05
**p< .01
***p< .001

SECTION 6 The reason I try to learn mathematics is...	Very true	Sort of true	Not very true	Not at all true	Fall mean	Spring mean	Signi-ficant t
29. To help me think more clearly in general.	1	2	3	4	2.15	2.27	
30. It's required for my program.	1	2	3	4	1.48	1.23	
31. I want to do well in the course.	1	2	3	4	1.26	1.16	
32. It's interesting.	1	2	3	4	2.26	2.00	
33. I'll get in trouble if I don't.	1	2	3	4	2.93	2.73	
34. I feel stupid if I can't understand something.	1	2	3	4	1.89	1.96	
35. I don't want to look dumb.	1	2	3	4	2.30	2.23	
36. To make the teacher think I'm a good student.	1	2	3	4	2.70	2.73	

SECTION 7

Instructions: Circle the number in front of your answer.

37. I am a...
    1. FR - 3
    2. SO - 10
    3. JR - 14
    4. SR - 0

38. I am a...
    1. Female - 22
    2. Male - 5

39. My current GPA is about...	Fall	Spring
1. 3.5 - 4.0	8	8
2. 3.0 - 3.5	8	7
3. 2.5 - 3.0	10	11
4. 2.0 - 2.5	1	0
5. 1.5 - 2.0	0	0

40. In high school, my math grade was usually a...
    1. F
    2. D
    3. C
    4. B
    5. A

Fall mean	Spring mean
4.13	4.19

41. Compared to other students in mathematics ability I'm...
    1. In the top 10%
    2. Above average
    3. About average
    4. Below average
    5. In the bottom 10%

2.5	2.5

42. Compared to how hard other students work at mathematics, I'm...

	Fall mean	Spring mean
1. In the top 10%		
2. Above average		
3. About average	2.70	2.69
4. Below average		
5. In the bottom 10%		

43. How important do you think it is to do well in math?

	Fall mean	Spring mean
1. Very important		
2. Sort of important		
3. Not very important	1.48	1.37
4. Not important at all		

SECTION B

Instructions: Answer each of the following questions in a sentence or two.

44. Do you think mathematicians work alone on problems or together?
45. Are the different mathematics courses you've taken (algebra, geometry, trig.) related to each other in any way or are they completely separate areas?
46. How much of your ability to do math shows up when you take math tests?
47. What can you do if you get stuck while doing a math problem?
48. In what way, if any, is the math you've studied useful?
49. Do you think that students can discover mathematics on their own, or does it have to be shown to them. Explain.
50. If you understand the material, how long should it take to solve a typical homework problem? What is a reasonable amount of time to work on a problem before you know it's impossible?
51. How can you know whether you understand something in math?
52. How important is the ability to memorize in learning mathematics? If anything else is important, explain how.

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